**FB1:** Find the remainder *r* (between 0 and 8) that we get when we divide by 9. Be sure to show your work.

From the problem above, a congruence equation can be written down as

The value *x* can then be found by starting with

and squaring both sides multiple times modulo 9 like this:

Additionally, the power of 7 in the question is 96, which means that *x* can be found by multiplying already found values of powers of 7 like this:

This shows that the remainder is 1. It can also be explicitly stated as

for some integer *k*. Then can be expressed as

the term being a multiple of 9, showing that the right-hand side has a remainder of 1 when divided by 9 and that also has a remainder of 1 when divided by 9.

**FB2:** Does the congruence equation (mod 25) have a solution for *x*? If it does, find the solution. If it does not, prove that it does not.

The congruence equation does have a solution because , which divides 7. This is proven by Euclid’s algorithm:

To find *x*, the highest common factor must be written out as

for some integers *s* and *t*. Rearranging it, one gets

for which the congruence equation in modulo 25 is

because of 25*t* being a multiple of 25. When multiplying the congruence by 7, it is

By comparing this to the given congruence equation, it can be seen that

To get *s*, the first equation in the Euclidean algorithm can be rearranged to make

making and

Therefore,

This can also be written as

for some integer *n*.

**FB3**: Find the acute angle between the lines and .

By rewriting the first equation as

and the second equation as

and looking at the gradients, two arbitrary vectors can be made:

each of which belongs to line 1 and line 2 respectively. Their magnitude and positive/negative direction do not matter because the angle is not dependent on either.

Let be the angle between the vectors ***a*** and ***b***. Then the angle is

Then the acute angle between vectors ***a*** and ***b*** is